INTRODUCTION

This volume offers an introduction to some basic mathematical and physical tools as well as methods from mathematics and physics required to follow the recent developments in several active topics at the interface between geometry and quantum field theory including duality, gauge field theory, geometric quantization, Seiberg-Witten theory, spectral properties and families of Dirac operators, and the geometry of loop groups.

It is based on lectures delivered during a Summer School "Geometric methods for Quantum Field Theory" held at Villa de Leyva, Colombia in July 1999, complemented by some short communications by participants of the school.

Since these lecture notes are aimed at beginning graduate students in physics or mathematics, the chapters are organized in an order of gradually increasing level of difficulty. Each chapter is self-contained and can be read independently; references to other chapters are made only to help the reader relate the different chapters.

The volume starts with an introductory course by Tilmann Wurzbacher on differentiable manifolds and symplectic geometry, followed by a presentation by Oussama Hijazi of the Dirac operator and its spectral properties, the Dirac operator playing a fundamental part in the sequel. The third lecture by Edwin Langmann offers an introduction to second quantization of fermions, pointing out along the way how the one dimensional Dirac operator arises in the quantization of loops. The Dirac operator on the circle is the main object of the fourth lecture by Krzysztof Wojciechowski who investigates its zeta-determinant. Zeta-determinants are defined using regularization techniques which are described in their geometric context by Sylvie Paycha in the fifth lecture. Using some of the geometric tools introduced in the first two lectures, in the sixth one, Tsou Sheung Tsun gives a concise introduction to gauge theory and a glimpse into duality via the electro-magnetic model. The last lecture by Hernan Ocampo, starting from electro-magnetic duality, leads the reader into the realm of Seiberg-Witten theory.

The short communications at the end of the volume, the topics of which are closely related to those of the preceding lectures, were selected by way of a strict refereeing procedure. We thank the referees most warmly for the wonderful job they did dedicating a lot of their time to help improve the short communications published here.

We are indebted to various organizations for their financial support. Let us thank first of all the French organization "C.I.M.P.A.". We are specially grateful to its former director Claude Lobry who showed his constant support during the preparation of the school and who was kind enough to accept our invitation to attend part of it. We also thank ECOS-Nord, this school being part of a long term scientific program between the Université Blaise Pascal in Clermont-Ferrand and the Universidad de Los Andes in Bogotá in the areas of mathematics and physics. We are also grateful to the Universidad de Los Andes, which was our main source of financial support in Colombia. In addition we are indebted to the I.C.T.P., "Banco de la República", "Academia Colombiana de Ciencias", I.C.E.T.E.X, and I.C.F.E.S. who also contributed to the financial support needed for this school.

Special thanks to Sergio Adarve (Universidad de Los Andes) coorganizer of the school who dedicated much time and energy to make this school possible in a country like Colombia where many difficulties are bound to arise along the way due to social, political and economic problems. Let us also thank Daniel Bennequin most warmly for his scientific advice when preparing the program of the school. We would furthermore like to express our gratitude to Rolando Roldan (Universidad de los Andes), coorganizer, and Jose Rafael Toro (Universidad de los Andes), whose contribution and support was essential for the success of the school.

We also thank Monica Vargas most warmly for doing a wonderful job for the practical organization of the school. Special thanks to Andrea Bernal, Andres Garcia, Marta Kovacsics, Diego Laverde and Alexandra Parra. Without all the people named here, all of whom helped with the organization in some way or another, before, during and after the school, this scientific event would not have left such vivid memories in the lecturers' and the participants' minds. Last but not least, thanks to all the participants who gave us all, lecturers, contributors of short communications and editors, the impulse to prepare this volume through the enthusiasm they showed us during the school.

We hope that these lectures will give -as much as the school itself seems to have given- young students the desire to pursue what might be their first acquaintance with some of the problems on the edge of mathematics and physics presented in this volume. On the other hand, we hope that the more advanced reader will find some pleasure reading about different outlooks on related topics and seeing how well-known geometric tools prove to be very useful in some areas of quantum field theory.

The Editors

Hernan Ocampo, Sylvie Paycha, Andres Reyes.

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