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Higher Dimensional Gravity, Propagating Torsion and AdS Gauge Invariance

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The most general theory of gravity in d dimensions which leads to second order field equations for the metric has $[(d-1)/2]$ free parameters. It is shown that requiring the theory to have the maximum possible number of degrees of freedom, fixes these parameters in terms of the gravitational and the cosmological constants. In odd dimensions, the Lagrangian is a Chern-Simons form for the (A)dS or Poincaré groups. In even dimensions, the action has a Born-Infeld-like form.

Torsion may occur explicitly in the Lagrangian in the parity-odd sector and the torsional pieces respect local (A)dS symmetry for $d = 4k-1$ only. These torsional Lagrangians are related to the Chern-Pontryagin characters for the (A)dS group. The additional coefficients in front of these new terms in the Lagrangian are shown to be quantized.

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I. INTRODUCTION

The possibility that spacetime may have more than four dimensions is now a standard assumption in high energy physics.. This underscores the need to critically examine the minimal requirements for a consistent theory of gravity in any dimension, including both general covariance and second order field equations for the metric. Although many different approaches have been followed in the generalizations to $d > 4$, most models assume the simplest generalization of General Relativity to higher dimensions, namely the Einstein-Hilbert action. The most general action for the metric satisfying the criteria of general covariance and second order field equations for $d > 4$ is a polynomial of degree $[d/2]$ in the curvature[1], the Lanczos-Lovelock **(LL)** theory. The LL theory in fact refers to a family parametrized by a set of real coefficients α_p, $p = 0, 1, ..., [d/2]$, which are not fixed from first principles.

In this note, it is shown these parameters are fixed in terms of the gravitational and the cosmological constants, through the requirement that the theory possess the largest possible number of degrees of freedom. As a consequence, the action in even dimensions has a Born-Infeld-like form, while in odd dimensions, the Lagrangian is a Chern-Simons form for the (A)dS or Poincaré groups. The same requirement implies that torsion may occur explicitly in the Lagrangian only for $d = 4k-1$. Each of these torsional Lagrangians are related to the Chern-Pontryagin characters for the (A)dS group, and the coefficients in front of them (β's) are shown to be quantized.

Here we adopt the first order approach, where the independent dynamical variables are the vielbein (e^a) and the spin connection (ω^{ab}), which obey first order differential field equations[2]. The standard second order form can be obtained if the torsion equations are solved for the connection and eliminated in favor of the vielbein – this step however, cannot be taken in general because the equations for ω^{ab} are not invertible for dimensions higher than four. The first order formalism has the added advantage that it can be expressed entirely in terms of differential forms and their exterior derivatives, without ever introducing the inverse vielbein or the Hodge $*$-dual.

In the next section we review extensions of gravity theory beyond the Einstein-Hilbert action for dimensions greater than four (LL theory). In Section III it is shown that in order to have the maximum possible number of degrees of freedom, the α_p's must be fixed in terms of the gravitational and the cosmological constants. In Section IV the inclusion of torsion explicitly in the Lagrangian is explored. New torsional Lagrangians are found, which are related to the Chern-Pontryagin characters for the (A)dS group in $d = 4k-1$ dimensions. Section V contains the discussion and summary.

II. BEYOND THE EINSTEIN-HILBERT ACTION

Assuming the spacetime geometry as given by the Einstein-Hilbert (**EH**) action –with or without cosmological constant– is the most reasonable choice in dimensions three and four[3], but not necessarily so for $d > 4$. The idea that a more general theory could be employed to describe

[1] Here $[x]$ represents the integer part of x.

[2] The curvature and torsion two-forms are related to e^a, and ω^{ab} through $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$, and $T^a = de^a + \omega^a{}_b \wedge e^b$, respectively.

[3] In $1+1$ dimensions, in order to write an action principle it is necessary supply the theory with an extra scalar field [1,2].

the spacetime geometry in dimensions larger than four – even in the absence of torsion– was first considered some sixty years ago by Lanczos [3]. More recently, it was observed that the low energy effective Lagrangian for gravity obtained from string theory would have curvature-squared terms –and higher powers as well– [4]. These terms are potential sources of inconsistencies as they would in general give rise to fourth order field equations and bring in ghosts. However, it was soon pointed out by Zwiebach [5] and Zumino [6], that if the effective Lagrangian would contain the higher powers of curvature in particular combinations, only second order field equations are produced and consequently no ghosts arise. The effective Lagrangian obtained by this argument, was precisely of the form proposed by Lanczos for $d=5$ and, for general d, by Lovelock [7].

In a more recent context, there are further clues that point in this direction. It is expected that the low energy regime of M-theory should be described by an eleven-dimensional supergravity of new type, with off-shell local supersymmetry [8], whose Lagrangian should contain higher powers of curvature [9]. A family of supergravity theories that satisfy these conditions has been proposed in [10–12], and the purely gravitational sector of those theories is an extension of ordinary EH gravity, as described below.

A. First Order Formalism

In standard General Relativity, the metric is viewed as the fundamental field, while the affine structure of spacetime (connection) is assumed to be a derived concept. The link between the two structures is the vanishing of the torsion tensor, which is often imposed as an identical, off-shell requirement for the connection. Consequently, the spin connection has no independent, propagating degrees of freedom and the spacetime torsion is not dynamically determined but constrained by fiat.

A purely metric formulation would be insufficient for the description of spinor fields because they couple the antisymmetric part of the affine connection, and therefore they are sources for the torsion. Hence, in a theory with fundamental spinors coupled to gravity, it is necessary that the metric and affine properties of spacetime be treated separately. Moreover, considering the fact that spinors provide a basis of irreducible representations for $SO(d-1,1)$ (Lorentz$_d$), but not for $GL(d)$, they must be defined on a local frame on the tangent space rather than in relation to a coordinate system on the base manifold.

Thus, in a theory containing fermions, it is more natural to look for a formulation of gravity in which ω^{ab} and e^a are dynamically independent, with curvature and torsion standing on similar footing. The first order formalism offers exactly this possibility. Indeed, when torsion is not set equal to zero, the standard variational principles –first, second and 1.5 order– are no longer necessarily equivalent. For example, varying the action with respect to e^a – in the "1.5 order formalism" [13], yields

$$\delta I = \frac{\delta I}{\delta e^a}\delta e^a + \frac{\delta I}{\delta \omega^{bc}}\frac{\delta \omega^{bc}}{\delta e^a}\delta e^a,$$

which would reduce to the Einstein equations (second order formalism) provided $\frac{\delta I}{\delta \omega^{bc}} = 0$. Thus, in particular, in the presence of spinors these formalisms would be different because in the second order case, the torsion equation is imposed as a constraint, but this is in fact a matter of choice.

In three and four dimensions, allowing ω^{ab} and e^a to be dynamically independent does not modify the standard picture in practice because any occurrence of torsion in the action leads to torsion-free classical solutions (in vacuum)[4]. In higher dimensions, however, theories that include torsion explicitly in the Lagrangian, cannot be related, even perturbatively, to their torsion-free counterparts [14].

B. Higher Dimensional Gravity: Lanczos-Lovelock Theory

The main fundamental assumptions in standard General Relativity are the requirements of general covariance and that the field equations for the metric be second order. Based on the same principles, the LL Lagrangian is defined as the most general d-form invariant under local Lorentz transformations, constructed with the vielbein, the spin connection, and their exterior derivatives, without using the Hodge dual[5] [6,15,16].

The LL Lagrangian is a polynomial of degree $[d/2]$ in the curvature,

$$I_G = \int \sum_{p=0}^{[d/2]} \alpha_p L^{(p)}, \tag{1}$$

where α_p are arbitrary constants, and

$$L^{(p)} = \epsilon_{a_1\cdots a_d} R^{a_1 a_2} \cdots R^{a_{2p-1}a_{2p}} e^{a_{2p+1}} \cdots e^{a_d}. \tag{2}$$

Here and in the sequel wedge product of forms is implicitly understood.

The first two terms in (1) are the EH action. Although General Relativity is contained in the LL action as a particular case, theories with higher powers of curvature are dynamically very different from EH, whose classical solutions are not even perturbatively related to those of

[4] In the case of coupling to spinning matter, the torsion equations allow expressing ω^{ab} in terms of e^a and the matter fields.

[5] Avoiding the Hogde dual guarantees that the fields ω^{ab} and e^a that extremize the action obey first order equations.

Einstein's theory. However, to lowest order in perturbation theory around a flat, torsion-free background, all of the $L^{(p)}$'s defined in (2) with $p \geq 2$ are total derivatives [6].

The $[(d+1)/2]$ dimensionful constants α_p in the LL action contrast with the two constants of the EH theory (G and Λ). In the following section the α_p's are selected according to the criterion that the integrability (or consistency) conditions for the field equations should not impose additional algebraic constraints on the curvature and torsion tensors. This guarantees that the fields attain the maximum number of degrees of freedom allowed by the spacetime dimension.

III. CONSISTENCY OF THE FIRST ORDER FORMALISM

Consider the LL action (1), as a functional of the spin connection and the vielbein, $I_G = I_G[\omega^{ab}, e^a]$. Varying with respect to these fields, the following field equations are obtained,

$$\delta e^a \to \mathcal{E}_a = 0, \tag{3}$$

$$\delta \omega^{ab} \to \mathcal{E}_{ab} = 0, \tag{4}$$

where we have defined

$$\mathcal{E}_a := \sum_{p=0}^{[\frac{d-1}{2}]} \alpha_p (d-2p) \mathcal{E}_a^p, \tag{5}$$

$$\mathcal{E}_{ab} := \sum_{p=1}^{[\frac{d-1}{2}]} \alpha_p p (d-2p) \mathcal{E}_{ab}^p, \tag{6}$$

and

$$\mathcal{E}_a^p \equiv \epsilon_{ab_1 \cdots b_{d-1}} R^{b_1 b_2} \cdots R^{b_{2p-1} b_{2p}} e^{b_{2p+1}} \cdots e^{b_{d-1}}, \tag{7}$$

$$\mathcal{E}_{ab}^p \equiv \epsilon_{aba_3 \cdots a_d} R^{a_3 a_4} \cdots R^{a_{2p-1} a_{2p}} T^{a_{2p+1}} e^{a_{2p+2}} \cdots e^{a_d}. \tag{8}$$

The $(d-1)$-forms $\mathcal{E}_a$ and $\mathcal{E}_{ab}$ are independent Lorentz tensors with the same number of components as the fields e^a and ω^{ab}, respectively. If there were algebraic relations among these tensors, so that (3) and (4) would not be independent, then the fields ω^{ab} and e^a would not be completely determined by their field equations and initial conditions. On the other hand, it is easy to check that by virtue of the Bianchi identities ($DR^{ab} = 0$, $DT^a = R^a_{\ b} e^b$), the following relations hold

$$D\mathcal{E}_a^p = (d-2p-1) e^b \mathcal{E}_{ba}^{p+1}, \tag{9}$$

for $0 \leq p \leq [(d-1)/2]$, which leads to the following off-shell identity

$$D\mathcal{E}_a \equiv \sum_{p=1}^{[\frac{d+1}{2}]} \alpha_{p-1} (d-2p+2)(d-2p+1) e^b \mathcal{E}_{ba}^p, \tag{10}$$

Which by consistency with (3) must also vanish. Moreover, taking the exterior product of (6) with e^b gives

$$e^b \mathcal{E}_{ba} \equiv \sum_{p=1}^{[\frac{d-1}{2}]} \alpha_p p (d-2p) e^b \mathcal{E}_{ba}^p, \tag{11}$$

which vanishes by virtue of (4).

Comparing the last two identities, one can see that if the coefficients α_p were generic, equations (3) and (4) would imply in general additional restrictions of the form $e^b \mathcal{E}_{ba}^p = 0$ for some p's. This in turn would imply that some components of the torsion tensor must vanish, freezing out some degrees of freeom in the theory, and at the same time, leaving other components of the curvature and torsion tensors are left undetermined by the field equations. Thus, different choices of α_p's correspond, in general, to theories with different numbers of physical degrees of freedom depending on how many additional off-shell constraints are imposed on the geometry by the last identities.

As we show next, among all the possible choices, there is a very special one which occurs only in odd dimensions, for which there are no additional constraints. In even dimensions, this possibility does not exist; in fact, equations (10) and (11) are proportional to each other term by term for $d = 2n-1$ but for $d = 2n$, both equations have different number of terms. We will treat each case separately.

A. $d = 2n-1$: Local (A)dS Chern-Simons Gravity

In odd dimensions, equations (10) and (11) have the same number of terms because the last term in (10) vanishes. Thus, if equations (10) and (11) are to imposse no further algebraic constraints on R^{ab} and T^a, the two series $D\mathcal{E}_a$ and $e^b \mathcal{E}_{ba}$ must be proportional term by term:

$$\gamma \alpha_{p-1} (d-2p+2)(d-2p+1) e^b \mathcal{E}_{ba}^p = \alpha_p p (d-2p) e^b \mathcal{E}_{ba}^p,$$

which implies the following recursion relation for the α_p's

$$\gamma \frac{\alpha_p}{\alpha_{p+1}} = \frac{(p+1)(d-2p-2)}{(d-2p)(d-2p-1)} \tag{12}$$

where $0 \leq p \leq n-1$, and γ is an arbitrary constant of dimension [length^2]. The solution of this equation is

$$\alpha_p = \alpha_0 \frac{(2n-1)(2\gamma)^p}{(2n-2p-1)} \begin{pmatrix} n-1 \\ p \end{pmatrix}. \tag{13}$$

Thus, the action contains only two fundamental constants, α_0 and γ, related to the gravitational and the cosmological constants through[6]

$$\alpha_0 = \frac{\kappa}{dl^{d-1}},$$
$$\gamma = -sgn(\Lambda)\frac{l^2}{2}. \tag{14}$$

Choosing the coefficients α_p as in (13), implies that the action is invariant not only under standard local Lorentz rotations ($\delta e^a = \lambda^a_{\ b} e^b$ and $\delta\omega^{ab} = -D\lambda^{ab}$), but also under local AdS boosts,

$$\delta e^a = -D\lambda^a$$
$$\delta\omega^{ab} = \frac{1}{l^2}(\lambda^a e^b - \lambda^b e^a). \tag{15}$$

This can be seen because the Lagrangian in (1) with the choice of coefficients (13) is the Euler-Chern-Simons form for $SO(d-1,2)$, that is, its exterior derivative is the Euler form in $2n$ dimensions E_{2n},

$$dL^{AdS}_{G\,2n-1} = \frac{\kappa l}{2n}\epsilon_{A_1\cdots A_{2n}}\bar{\mathbf{R}}^{A_1A_2}\cdots\bar{\mathbf{R}}^{A_{2n-1}A_{2n}}$$
$$= \bar{\kappa}\mathsf{E}_{2n}, \tag{16}$$

where

$$\bar{\mathbf{R}}^{AB} = \begin{bmatrix} R^{ab} + \frac{1}{l^2}e^a e^b & T^a/l \\ -T^b/l & 0 \end{bmatrix}, \tag{17}$$

defines the Lie algebra valued AdS curvature $\mathbf{F} = \frac{1}{2}\bar{\mathbf{R}}^{AB}J_{AB} = d\mathbf{A} + \mathbf{A}^2$ in terms of the AdS connection $\mathbf{A} = \frac{1}{2}\mathbf{W}^{AB}J_{AB} = \frac{1}{2}\omega^{ab}J_{ab} + e^a J_{ad+1}$ [18,19]. Hence, equations (3) and (4) can be cast as different components of a single AdS covariant equation

$$\delta\mathbf{W}^{AB} \to \mathcal{E}_{AB} := \epsilon_{ABA_3\cdots_{d+1}}\bar{\mathbf{R}}^{A_3A_4}\cdots\bar{\mathbf{R}}^{A_dA_{d+1}} = 0, \tag{18}$$

which transforms as a tensor under local AdS gauge transformations which include (15), $\delta\mathbf{W}^{AB} = -\nabla\lambda^{AB}$, where ∇ is the exterior covariant derivative in the AdS connection. Considering this, the consistency condition $e^b\mathcal{E}_{ba} = \gamma D\mathcal{E}_a$ does not produce additional constraints, because it is just a component of the identity

$$\nabla\mathcal{E}_{AB} \equiv 0, \tag{19}$$

which is trivially satisfied by virtue of the AdS Bianchi identity, $\nabla\bar{\mathbf{R}}^{AB} = 0$.

[6] For any dimension, l is a length parameter related to the cosmological constant by $\Lambda = \pm\frac{(d-1)(d-2)}{2l^2}$. In the following we will choose the negative sign, but the analysis does not depend on it. Here, the gravitational constant G is related to κ through $\kappa^{-1} = 2(d-2)!\Omega_{d-2}G$ (see Ref. [17]).

B. $d = 2n$: Born-Infeld-Like Gravity

For even dimensions, equation (10) has one more term than (11). Therefore, both series cannot be compared term by term and one must follow a different route. It can be noted that equation (4) is an exterior covariant derivative,

$$\mathcal{E}_{ab} = D\mathcal{T}_{ab}, \tag{20}$$

where,

$$\mathcal{T}_{ab} := \frac{\delta L}{\delta R^{ab}} = \sum_{p=1}^{[\frac{d-1}{2}]}\alpha_p p\mathcal{T}^p_{ab}, \tag{21}$$

and

$$\mathcal{T}^p_{ab} = \epsilon_{aba_3\cdots a_d}R^{a_3a_4}\cdots R^{a_{2p-1}a_{2p}}e^{a_{2p+1}}\cdots e^{a_d}. \tag{22}$$

Note also that $\mathcal{T}^p_{ab}$ is related with $\mathcal{E}^p_a$ and $\mathcal{E}^p_{ab}$ through

$$e^b\mathcal{T}^p_{ab} = \mathcal{E}^{p-1}_a, \tag{23}$$

$$D\mathcal{T}^p_{ab} = (d-2p)\mathcal{E}^p_{ab}, \tag{24}$$

for $1 \leq p \leq [\frac{d-1}{2}]$.

Differentiating both sides of (23) and using (24), identity (10) can also be written for $d = 2n$ as

$$D\mathcal{E}_a = T^b\sum_{p=1}^{n-1}2\alpha_{p-1}(n-p+1)\mathcal{T}^p_{ab}$$
$$-\sum_{p=1}^{n-1}4\alpha_{p-1}(n-p+1)(n-p)e^b\mathcal{E}^p_{ba}. \tag{25}$$

This equation can be compared with the second identity (11)

$$e^b\mathcal{E}_{ba} \equiv \sum_{p=1}^{n-1}2p\alpha_p(n-p)e^b\mathcal{E}^p_{ba}. \tag{26}$$

Both (20) and (26) can be zero either if $T^a = 0$, or $\mathcal{T}_{ab} = 0$. However, those are excessive conditions for the vanishing of (25). It is sufficient to impose the weaker conditions $T^a\mathcal{T}_{ab} = 0$, and at the same time that the second term in (25) be proportional to the series in (26). Now, both series possess the same number of terms, and therefore the solution which allows the maximum number of degrees of freedom is the one for which both series are equal term by term, up to a global factor. Hence, one obtains the following recursion relation for the α_p's:

$$2\gamma(n-p+1)\alpha_{p-1} = p\alpha_p, \tag{27}$$

for some fixed γ. With this relation, equation (25) reads,

$$D\mathcal{E}_a = \frac{1}{\gamma}(T^b \mathcal{T}_{ab} - e^b \mathcal{E}_{ab}) = 0, \tag{28}$$

and therefore it is apparent that if T^a is just a null vector of $\mathcal{T}_{ab}$, both consistency conditions are the same.

The solution of the recursion relation (27) is

$$\alpha_p = \alpha_0 (2\gamma)^p \begin{pmatrix} n \\ p \end{pmatrix}, \tag{29}$$

with $0 \leq p \leq n-1$. This formula can be extended to $p = n$ at no extra cost, because it only amounts to adding the Euler density to the Lagrangian with the weight $\alpha_n = \alpha_0 (2\gamma)^n$.

The action depends only on the gravitational and the cosmological constants, as in odd dimensions, given by (14). The choice of coefficients (29) implies that the Lagrangian takes the form

$$L = \frac{\kappa}{2n} \epsilon_{a_1 \cdots a_d} \bar{R}^{a_1 a_2} \cdots \bar{R}^{a_{d-1} a_d}, \tag{30}$$

which is the pfaffian of the 2-form $\bar{R}^{ab} = R^{ab} + \frac{1}{l^2} e^a e^b$, and can be formally written as the Born-Infeld **(BI)**-like form [20],

$$L = 2^{n-1}(n-1)! \kappa \sqrt{\det \left(R^{ab} + \frac{1}{l^2} e^a e^b \right)}. \tag{31}$$

In four dimensions (31) reduces to a particular linear combination of the Einstein-Hilbert action, the cosmological constant and the Euler density. Although this last term does not contribute to the field equations, it plays an important role in the definition of conserved charges for gravitation theories in dimensions $2n \geq 4$ [21,22,17].

The field equations (3) and (4), now take the form

$$\begin{aligned} \delta e^a &\rightarrow \epsilon_{a b_1 \cdots b_{d-1}} \bar{R}^{b_1 b_2} \cdots \bar{R}^{b_{d-3} b_{d-2}} e^{b_{d-1}} = 0, \\ \delta \omega^{ab} &\rightarrow \epsilon_{ab a_3 \cdots a_d} \bar{R}^{a_3 a_4} \cdots \bar{R}^{a_{d-3} a_{d-2}} T^{a_{d-1}} e^{a_d} = 0. \end{aligned} \tag{32}$$

One could consider sectors of the phase space for which

$$\mathcal{T}_{ab} = \frac{\kappa}{2} \epsilon_{ab a_3 \cdots a_d} \bar{R}^{a_3 a_4} \cdots \bar{R}^{a_{d-1} a_d} = 0, \tag{33}$$

which solves the field equations (32) identically without requiring $T^a = 0$.

The two-form $\bar{R}^{ab}$ is a piece of the AdS curvature (17). This fact seems to suggest that the system might be naturally described in terms of an AdS connection (see, e.g., [23]). However, that is incorrect: in even dimensions, the Lagrangian (30) is invariant under local Lorentz transformations and *not* under the entire AdS group. In contrast, as shown above, it is possible to construct gauge invariant theories of gravity under the full AdS group in odd dimensions.

C. Dynamical Behavior

As shown above, unlike in the EH theory, the field equations of BI and CS theories do not imply the vanishing of torsion in absence of matter. On the contrary, assuming $T^a = 0$ as a constraint links the transformation of the spin connection with that of the vielbein,

$$\delta \omega^{ab} = \frac{\delta \omega^{ab}}{\delta e^c} \delta e^c. \tag{34}$$

This dynamical dependence between ω^{ab} and e^a, spoils the possibility of interpreting the local AdS boosts – or local translational invariance, in the $\Lambda \rightarrow 0$ limit – as a gauge symmetry of the action. Thus, the spin connection and the vielbein –the soldering between the base manifold and the tangent space– cannot be identified as the compensating fields for local Lorentz rotations and AdS boosts, respectively. Hence, gravitation can be realized as a truly gauge theory with fiber bundle structure, where ω^{ab} and e^a are connection fields only if the torsion is not fixed to zero. As shown above, this possibility is fully realized in odd dimensions for the CS Lagrangian only.

For a generic LL theory, when torsion is set equal to zero, the number of degrees of freedom is the same as in the EH theory, namely $\frac{d(d-3)}{2}$ [16]. These degrees of freedom correspond to the components of the vielbein that remain after fixing the local Lorentz and diffeomorphism invariances. On the other hand, CS theory in $d = 2n-1$ without assuming vanishing torsion has $(n-1)(2n^2 - 5n + 1)$ additional degrees of freedom [24]. These extra local degrees of freedom cannot be excitations described by the vielbein, because the gauge symmetry of the theory can be used to gauge away $\frac{d(d+3)}{2}$ of its components, just like in the non-CS case. Hence, the extra degrees of freedom must be carried by the contorsion tensor $k^{ab} := \omega^{ab} - \bar{\omega}^{ab}(e)$, where $\bar{\omega}^{ab}(e)$ is the solution of $T^a = 0$.

In view of the preceding analysis, there seems to be no reason to exclude torsion from the Lagrangian itself. In the next section we explore the possibility of adding torsion explicitly to the action.

IV. ADDING TORSION EXPLICITLY IN THE LAGRANGIAN

The generalization of the Lanczos-Lovelock theory to include torsion explicitly is obtained assuming the Lagrangian to be the most general d-form constructed with the vielbein and the spin connection without using the Hodge dual, and invariant under local Lorentz transformations. A constructive algorithm to produce all possible local Lorentz invariants from e^a, R^{ab} and T^a is given in Ref. [14]. As with the LL Lagrangian, the explicit inclusion of torsional terms brings in a number of arbitrary dimensionful coefficients β_k analogous to the α_p's.

The aim of this section is to show that in certain dimensions the β's can be suitably chosen so as to enlarge the local Lorentz invariance into the AdS gauge symmetry[7].

If no additional structure (e.g., inverse metric, Hodge dual $(*)$, etc.) is assumed, AdS invariant integrals can only be produced in $4k$ and $4k-1$ dimensions. This can be seen as follows: As is well-known (see, e.g., [25]), in $2n$ dimensions, the only $2n$-forms invariant under $SO(N)$ constructed in terms of the $SO(N)$ curvature are the Euler density –for $N = 2n$ only–, and the n-th Chern characters –for any N. An important difference between these invariants is that the Euler form is even under parity transformations, while the latter is odd. Parity is defined by the sign change induced by a simultaneous inversion of one coordinate in the tangent space and in the base manifold. Thus, for instance the Euler density, $\mathsf{E}_{2n} = \epsilon_{a_1\cdots a_{2n}} R^{a_1a_2} \cdots R^{a_{2n-1}a_{2n}}$, is even under parity, while the Lorentz Chern classes, $R^{a_1}_{\ b_2} \cdots R^{a_{2n}}_{\ a_1}$, and the torsional invariants such as $e_a R^a_{\ b} T^b$ are parity odd.

In the previous section we discussed all possible Lagrangians of the form $\epsilon[R]^p[e]^{d-2p}$. In what follows we concentrate on the construction of the pure gravity sector as a gauge theory which is parity-odd. That sector is described by Lagrangians containing Lorentz-invariant products of the fields and their exterior derivatives, which do not contain the Lorentz Levi-Civita symbol $\epsilon_{a_1\cdots a_d}$. This construction was sketched through briefly in the context of supergravity in Refs. [26,10,12].

In even dimensions the only AdS-invariant d-forms are, apart from the Euler density, linear combinations of products of the type

$$\mathsf{P}_{r_1\cdots r_s} = \mathsf{C}_{r_1} \cdots \mathsf{C}_{r_s}, \tag{35}$$

with $2(r_1 + r_2 + \cdots + r_s) = d$. Here

$$\mathsf{C}_r = \mathrm{Tr}(\mathbf{F})^r, \tag{36}$$

defines the r^{th} Chern character of $SO(N)$. Now, since the curvature two-form $\mathbf{F}$ is in the vector representation it is antisymmetric in the group indices. Thus, the powers r_j in (36) are necessarily even, and therefore (35) vanishes unless d is a multiple of four. These results can be summarized in the following lemmas:

Lemma 1: For $d = 4k$, the only parity-odd d-forms built from e^a, R^{ab} and T^a, invariant under the AdS group, are the Chern characters for $SO(d+1)$.

Lemma 2: For $d = 4k+2$, there are no parity-odd $SO(d+1)$-invariant d-forms constructed from e^a, R^{ab} and T^a.

Since the expressions $\mathsf{P}_{r_1\cdots r_s}$ in (35) are $4k$-dimensional closed forms, they are at best boundary terms which do not contribute to the classical equations (although they would assign different phases to configurations with nontrivial torsion in the quantum theory). In view of this, it is clear why attempts to construct purely gravitational theories with local AdS invariance in even dimensions have proven unsuccessful in spite of several serious efforts [23,27].

The form $\mathsf{P}_{r_1\cdots r_s}$ can be expressed locally as the exterior derivative of a $(4k-1)$-form,

$$\mathsf{P}_{r_1\cdots r_s} = dL^{AdS}_{T\ 4k-1}(\mathbf{A}). \tag{37}$$

This implies that for each collection $\{r_1, \cdots r_s\}$, $L^{AdS}_{T\ 4k-1}$ is a good Lagrangian for the AdS group $(SO(4k))$ in $4k-1$ dimensions. In a given dimension, the most general Lagrangian of this sort is a linear combination of all possible $L^{AdS}_{T\ 4k-1}$'s. It can be directly checked that these Lagrangians necessarily involve torsion explicitly.

These results can be summarized in the following

Theorem: In odd-dimensional spacetimes, there are two families of first-order gravitational Lagrangians $L(e,\omega)$, invariant under local AdS transformations:

a: The Euler-Chern-Simons form $L^{AdS}_{G\ 2n-1}$, in $d = 2n-1$ [parity-even]. Its exterior derivative is the Euler density in $2n$ dimensions and does not involve torsion explicitly, and

b: The Pontryagin-Chern-Simons forms $L^{AdS}_{T\ 4k-1}$, in $d = 4k-1$ [parity-odd]. Their exterior derivatives are Chern characters in $4k$ dimensions and involve torsion explicitly.

It must be stressed that locally AdS-invariant gravity theories exist only in odd dimensions. They are *genuine* gauge systems, whose action comes from topological invariants in $d+1$ dimensions. These topological invariants can be written as the trace of a homogeneous polynomial of degree n in the AdS curvature. In summary, for dimensions $4k-1$ both **a-** and **b-**families exist, and for $d = 4k+1$ only the **a-** family appears

A. Examples for $d = 2n$

In $d = 4$, the only local Lorentz-invariant 4-forms constructed with the recipe just described are:

$$\mathsf{E}_4 = \epsilon_{abcd} R^{ab} R^{cd}$$
$$L_{EH} = \epsilon_{abcd} R^{ab} e^c e^d$$
$$L_\Lambda = \epsilon_{abcd} e^a e^b e^c e^d$$

[7] Here $SO(d)$ and $SO(d-1,1)$ will be used indistinguishably to represent the Lorentz group in d dimensions, while the d-dimensional (A)dS group will be denoted by $SO(d-1,2)$, $SO(d,1)$ or $SO(d+1)$, since the analysis that follows is insensitive to the signature.

$$\mathsf{C}_2 = R^a{}_b R^b{}_a$$
$$L_{T_1} = R^{ab} e_a e_b$$
$$L_{T_2} = T^a T_a.$$

The first three terms are even under parity and the rest are odd. Of these, E_4 and C_2 are topological invariant densities (closed forms): the Euler density and the second Chern character for $SO(4)$, respectively. The remaining four terms define the most general pure gravity action in four dimensions,

$$I = \int_{M_4} \left[\alpha L_{EH} + \beta L_\Lambda + \gamma L_{T_1} + \rho L_{T_2}\right]. \tag{38}$$

The first two terms can be combined with E_4 into the Born-Infeld form (31) which is locally invariant under Lorentz, but not under AdS. It can also be seen that if $\gamma = -\rho$, the last two terms are combined into a topological invariant density, the Nieh-Yan form [28]. This choice implies that the entire odd part of the action becomes a boundary term. Furthermore, C_2, L_{T_1} and L_{T_2} can be combined into the second Chern character of the AdS group,

$$R^a{}_b R^b{}_a + \frac{2}{l^2}(T^a T_a - R^{ab} e_a e_b) = \bar{R}^A_{\ B} \bar{R}^B_{\ A}. \tag{39}$$

The form (39) is the only AdS invariant constructed just with e^a, ω^{ab} and their exterior derivatives, and therefore there are no locally AdS-invariant gravity theories in four dimensions.

In view of Lemmas (1) and (2), the corresponding AdS-invariant functionals in higher dimensions can be written in terms of the AdS curvature as linear combinations of terms like

$$\tilde{I}_{r_1 \cdots r_s} = \int_M \mathsf{C}_{r_1} \cdots \mathsf{C}_{r_s}, \tag{40}$$

where $\mathsf{C}_r = \mathrm{Tr}[(\bar{R}^A_{\ B})^r]$ is the r-th Chern character for the AdS group, and $dim(M) = r_1 + \cdots + r_s$ is a multiple of four. For example, in $d = 8$ the Chern characters are

$$\mathsf{C}_4 = \mathrm{Tr}[(\bar{R}^A_{\ B})^4], \tag{41}$$

and

$$(\mathsf{C}_2)^2 = \mathrm{Tr}[(\bar{R}^A_{\ B})^2] \wedge \mathrm{Tr}[(\bar{R}^A_{\ B})^2]. \tag{42}$$

The corresponding integrals $\tilde{I}_4$ and $\tilde{I}_{2,2}$ are topological invariants that characterize the maps $SO(9) \to M_8$. Furthermore, as already mentioned, $\tilde{I}_{r_1 \cdots r_s}$ vanishes if one of the r's happens to be odd, which is the case in $4k+2$ dimensions. Thus, one concludes that there are no torsional AdS-invariant gauge theories for gravity in even dimensions.

B. Examples for $d = 2n - 1$

The simplest example occurs in three dimensions, where there are two locally AdS invariant Lagrangians, namely, the Einstein-Hilbert with cosmological constant,

$$L^{AdS}_{G\,3} = \frac{1}{l}\epsilon_{abc}[R^{ab}e^c + \frac{1}{3l^2}e^a e^b e^c], \tag{43}$$

and the "exotic" Lagrangian [29]

$$L^{AdS}_{T\,3} = L^*_3(\omega) + \frac{2}{l^2} e_a T^a, \tag{44}$$

where

$$L^*_3(\omega) \equiv \omega^a{}_b d\omega^b{}_a + \frac{2}{3}\omega^a{}_b \omega^b{}_c \omega^c{}_a. \tag{45}$$

The Lagrangians (43, 44, 45) are the Euler, the Pontryagin and the Lorentz Chern-Simons forms, respectively. The most general action for gravitation in $d = 3$, which is invariant under AdS is therefore the linear combination $\alpha L^{AdS}_{G\,3} + \beta L^{AdS}_{T\,3}$.

For $d = 4k - 1$, the number of possible exotic forms grows as the number of elements of the partition set $\pi(k)$ of k, in correspondence with the number of composite Chern invariants of the form:

$$\mathsf{P}_{\{r_j\}} = \prod_{r_j \in \pi(k)} \mathsf{C}_{r_j}. \tag{46}$$

Thus, the most general Lagrangian in $4k-1$ dimensions takes the form

$$\kappa L^{AdS}_{G\,4k-1} + \beta_{\{r_j\}} L^{AdS}_{T\,\{r_j\}\,4k-1}, \tag{47}$$

where $dL^{AdS}_{T\,\{r_j\}\,4k-1} = \mathsf{P}_{r_1 \cdots r_s}$, with $\sum_j r_j = 4k$. These Lagrangians are not boundary terms and, unlike the even-dimensional case, they have proper dynamics. For example, in seven dimensions one finds

$$L^{AdS}_{T\,7} = \beta_{2,2}[R^a{}_b R^b{}_a + 2(T^a T_a - R^{ab} e_a e_b)] L^{AdS}_{T\,3}$$
$$+ \beta_4 [L^*_7(\omega) + 2(T^a T_a + R^{ab} e_a e_b) T^a e_a + 4 T_a R^a{}_b R^b{}_c e^c],$$

where L^*_{2n-1} is the Lorentz-CS $(2n-1)$-form,

$$dL^*_{2n-1}(\omega) = \mathrm{Tr}[(R^a_{\ b})^n]. \tag{48}$$

It should be noted that the coefficients κ and $\beta_{\{r_j\}}$ are arbitrary and dimensionless. As is shown in the next section, these coefficients must be quantized by an extension of the argument used to prove that κ in (16) is also quantized [30]. We now extend that argument to show that the β's in (47) are also quantized.

C. Quantization of parameters

Consider the action for the connection $\mathbf{A}$ on a $(2n-1)$-dimensional, compact, oriented, simply connected manifold M without boundary, which is the boundary of an oriented $(2n)$-dimensional manifold Ω. By Stokes' theorem, the action for (47) can be written as an integral on Ω,

$$I_{\Omega}^{AdS}[\mathbf{A}] = \int_{\Omega} \left(\bar{\kappa}\mathsf{E}_{2n} + \beta_{\{r\}}\mathsf{P}_{\{r\}}\right). \tag{49}$$

(For $d = 4k+1$ the last term is absent as the $\mathsf{P}_{\{r\}}$'s vanish). $I_{\Omega}^{AdS}[\mathbf{A}]$ describes a topological field theory on Ω for $\mathbf{A}$ which should be insensitive to the change of Ω by another orientable manifold Ω' with the same boundary, i.e., $\partial\Omega = M = \partial\Omega$'. Thus we have

$$I_{\Omega}^{AdS}[\mathbf{A}] = I_{\Omega'}^{AdS}[\mathbf{A}] + \int_{\Omega\cup\Omega'} \left(\bar{\kappa}\mathsf{E}_{2n} + \beta_{\{r\}}\mathsf{P}_{\{r\}}\right), \tag{50}$$

where the orientation of Ω' has been reversed. Now, $\Gamma := \Omega\cup\Omega'$ is a closed oriented manifold formed by joining Ω and Ω' continuously along M. Then (50) can be written as

$$I_{\Omega}^{AdS}[\mathbf{A}] = I_{\Omega'}^{AdS}[\mathbf{A}] + \bar{\kappa}\chi[\Gamma] + \beta_{\{r_j\}}\mathsf{p}_{\{r_j\}}[\Gamma], \tag{51}$$

where $\mathsf{p}_{\{r\}} = \int_{\Gamma}\mathsf{P}_{\{r_j\}}$.

Substituting I_{Ω} by $I_{\Omega'}$ would have no effect in the path integral for the quantum theory provided the difference $\Delta[\Gamma] = I_{\Omega} - I_{\Omega'}$ is an integer multiple of Planck's constant h which, in addition, cannot change under continuous deformations of the fields. Thus, we have

$$\begin{aligned}\Delta[\Gamma] &= \bar{\kappa}\chi[\Gamma] + \beta_{\{r_j\}}\mathsf{p}_{\{r_j\}}[\Gamma] \\ &= mh.\end{aligned} \tag{52}$$

Now, since the Euler and the Pontryagin numbers $\chi[\Gamma]$ and $\mathsf{p}_{\{r_j\}}[\Gamma]$ are integers, the coefficients $\bar{\kappa}$ and $\beta_{\{r_j\}}$ are necessarily quantized.

The preceding argument is rigorously valid for a manifold with Euclidean signature. If M is locally Minkowskian one can apply the same reasoning to the analytic continuation of the path integral in which the base manifold M and its tangent bundle $T(M)$ are simultaneously Wick-rotated. This has the effect that the group of rotations on $T(M)$ may have nontrivial homotopy group, $\pi_{2n-1}[SO(2n)]$ so that the Chern characters can be nonzero.

V. DISCUSSION AND SUMMARY

1. Exact Solutions

It is apparent from the field equations for CS and BI theories (18,32), that any locally AdS spacetime is a solution for them. Apart from anti-de Sitter space itself, some interesting spacetimes of negative constant curvature are topological black holes of Refs. [31]. Furthermore, for each dimension, there is a unique static, spherically symmetric, asymptotically AdS black hole solution [19], as well as their topological extensions [32]. Similarly, Friedman-Robertson-Walker cosmologies have also been found [33].

Torsional AdS-invariant terms can be coupled only for CS theory in $4k-1$ dimensions. The contributions of these terms to the field equations, vanish identically for the solutions just mentioned –but not for all solutions.

2. Local AdS Symmetry and Geometric Equivalence

It may be stressed that the field equations in the CS case, with or without torsional terms, are manifestly locally AdS-covariant, which gives rise to a paradoxical situation: Under the action of an AdS transformation which is not contained in the Lorentz subgroup (15), the curvature and torsion tensors transform as:

$$\begin{aligned}\delta\bar{R}^{ab} &= \frac{1}{l^2}(\lambda^a T^b - \lambda^b T^a), \\ \delta T^a &= -\bar{R}^a_{\ b}\lambda^b.\end{aligned} \tag{53}$$

Thus, in general, a solution with non-vanishing AdS curvature and zero torsion can be mapped into another one with torsion. These solutions are not diffeomorphically equivalent to each other. In fact, the metric transforms under (15) as

$$\delta g_{\mu\nu} = \delta_\xi g_{\mu\nu} - \xi^\lambda e_{a(\nu}T^a_{\mu)\lambda}, \tag{54}$$

where δ_ξ stands for a diffeomorphism whose parameter satisfies $\lambda^a = e^a_\mu\xi^\mu$. This implies that in the presence of torsion, the new metric is in general not diffeomorphic to the old one. Furthermore, even if there is no torsion to begin with, by virtue of (54) the new metric will eventually be diffeomorphically inequivalent to higher order in λ^a.

At first glance, it would seem that these two solutions should be physically inequivalent; in fact, these solutions have different geodesic structure. The apparent paradox stems from the fact that the geodesic equation is Lorentz covariant, but not AdS covariant. The crucial point, is what one means by "physically equivalent". The situation is analogous to the transformation of the electromagnetic field under a Lorentz boost: $\mathbf{E}$ and $\mathbf{B}$ fields transform, and even if one start with a purely magnetic (electric) configuration, to second order in (v/c) one finds both.

3. String-induced Gravity

In the context of string theory, as shown in [5,6], the LL action is the only ghost free candidate for a low energy gravitational effective theory. This is a consequence of the fact that this are the only theories which gives rise second order field equations for the metric. On the other hand, it has been argued that the on-shell S-matrix is unchanged under metric redefinitions $g_{\mu\nu} \to g_{\mu\nu} + \alpha R_{\mu\nu} + \beta R$ [34], which changes completely the polynomial structure of the effective Lagrangian. This in general brings in ghosts, and worst yet, modifies the dynamical structure of the classical theory: the new field equations are fourth order, and they no longer pose a well defined Cauchy problem and spoils the causality features.

On the other hand, similar field redefinitions of the form $A_\mu \to A_\mu(A)$ are not acceptable in gauge theories since this would severely damage gauge invariance, spoiling essential features of the quantum theory. In the absence of a quantum theory of gravity, there seems to be no way to fix the form of the action completely unless the theory could be formulated as a gauge theory with fiber bundle structure. This is precisely the case for the CS gravity theories, whose simplest example occurs in $2+1$ dimensions. Although this gauge invariance of $2+1$ gravity is not always emphasized, it lies at the heart of the proof of integrability of the theory [29]. Higher dimensional CS actions have no dimensionful constants when written in terms of the AdS connection, so that the fields have canonical dimension 1 and the action describes a bona fide AdS gauge system. The corresponding quantum theory as well as its local supersymmetric extensions would be renormalizable by power counting and possibly finite [30].

4. Supergravity

The analysis of stability and positivity of the energy for these theories is a nontrivial problem. However some insights can be gained from the supersymmetric extension of the CS theory, for which, the expectation values of different charges should be related by Bogomolny'i bounds. The supersymmetric extension of gravity theories described here for $d = 4k - 1$ was discussed in [10], and in general for $d = 2n - 1$, in [11,12]. The resulting supergravity theories are locally invariant under the supersymmetric extensions of AdS, so that supersymmetry is realized in the fiber rather than on the base manifold[8]. A key point in that construction is that supersymmetry requires the inclusion of torsional terms from the start, which justifies considering such terms in the purely bosonic theory. The possible connection between the new eleven dimensional supergravity and M-Theory is an open problem, as can be readily inferred from a cursory review of Refs. [36–42].

The first example of a supergravity action containing the LL-action was worked out by Chamseddine in five dimensions [18]. This construction, however cannot be generalized to arbitrary higher odd dimensions unless torsional terms are introduced in the gravitational sector.

One can show that, around an appropriate background, the conserved charges satisfy a central extension of the super AdS algebra, which leads to a Bogomolny'i bound on the bosonic charges. As usual, solutions with Killing spinors saturate the bound[9].

In $d = 2n - 1$, one should expect the existence of a new kind of $2n$-dimensional theory at the boundary. That theory should be constructed on the generalization of the centrally extended gauge algebra, which can be viewed as the superconformal algebra at the boundary[10]. These theories should be a rich arena to test the recently conjectured AdS/CFT correspondence [43].

5. Summary

In sum, it was shown that requiring LL theories to have the maximum possible number of degrees of freedom, fixes their $[(d-1)/2]$ free parameters in terms of the gravitational and the cosmological constants. Following this criterion, the selected theories fall into two families. In even dimensions, torsion can be assumed to be a null vector of $\mathcal{T}_{ab}$ defined in (33), which is in general much weaker than imposing $T^a = 0$ by fiat, so that the resulting theory has a Born-Infeld form. In odd dimensions, torsion needs not be constrained at all in the theory, and the action can be written as a CS form. In that case, the vielbein and the spin connection can be viewed as different components of an (A)dS or Poincaré connection, so that its local symmetry is enlarged from Lorentz$_d$ to (A)dS$_d$ (or Poincaré when $\Lambda = 0$) [44].

The existence of propagating degrees of freedom – associated with the spacetime contorsion k^{ab}–, makes it natural to look at the parity -odd sector of the theory, which implies to consider torsional terms explicitly in the Lagrangian. The explicit inclusion of torsional terms brings in a number of new arbitrary dimensionful coefficients $\beta_{\{r\}}$ analogous to the α_p's in the LL Lagrangian. It was shown that β's can be suitably chosen so as to enlarge the local Lorentz invariance into the AdS gauge symmetry. In that case, torsion may occur explicitly in the Lagrangian in the parity-odd sector and for $d = 4k - 1$ only. These torsional Lagrangians are related to the Chern-Pontryagin characters for the (A)dS group.

[8] An exceptionally simple case occurs when the coefficients α_p in the theory are chosen so that the bosonic system is locally Poincaré invariant. The supersymmetric extension was constructed in Ref. [35].

[9] The five-dimensional case was analyzed partially in [36]

[10] We thank Marc Henneaux for fruitful discussions about this point.

Thus, for $d = 4k - 1$, the most general theory which allow the existence of independent propagating degrees of freedom for the contorsion, has a new set of parameters –the $\beta_{\{r\}}$'s–, which are shown to be quantized.

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$$\alpha_p = \frac{\alpha_0 d(2\gamma)^p}{(d-2p)} \begin{pmatrix} k \\ p \end{pmatrix},$$

which are determined requiring the existence of a unique maximally symmetric background solution. The Einstein-Hilbert action is recovered for $k=1$, and BI and CS Lagrangians correspond to $k=n-1$, and can be shown to possess well behaved black hole solutions for all k [17].